

Design of Engineering Experiments

Part 3 – The Blocking Principle

- Text Reference, Chapter 4
- **Blocking** and **nuisance factors**
- The randomized complete block design or the **RCBD**
- Extension of the ANOVA to the RCBD
- Other blocking scenarios...Latin square designs

The Blocking Principle

- **Blocking** is a technique for dealing with **nuisance factors**
- A **nuisance** factor is a factor that probably has some effect on the response, but it's of no interest to the experimenter...however, the variability it transmits to the response needs to be minimized
- Typical nuisance factors include batches of raw material, operators, pieces of test equipment, time (shifts, days, etc.), different experimental units
- **Many** industrial experiments involve blocking (or should)
- Failure to block is a common flaw in designing an experiment (consequences?)

The Blocking Principle

- If the nuisance variable is **known** and **controllable**, we use **blocking**
- If the nuisance factor is **known** and **uncontrollable**, sometimes we can use the **analysis of covariance** (see Chapter 15) to remove the effect of the nuisance factor from the analysis
- If the nuisance factor is **unknown** and **uncontrollable** (a **“lurking” variable**), we hope that **randomization** balances out its impact across the experiment
- Sometimes several sources of variability are **combined** in a block, so the block becomes an aggregate variable

The Hardness Testing Example

- Text reference, pg 120
- We wish to determine whether 4 different tips produce different (mean) hardness reading on a Rockwell hardness tester
- Gauge & measurement systems capability studies are frequent areas for applying DOX
- Assignment of the tips to an **experimental unit**; that is, a test coupon
- Structure of a completely randomized experiment
- The test coupons are a source of **nuisance variability**
- Alternatively, the experimenter may want to test the tips across coupons of various hardness levels
- The need for blocking

The Hardness Testing Example

- To conduct this experiment as a RCBD, assign all 4 tips to each coupon
- Each coupon is called a “**block**”; that is, it’s a more homogenous experimental unit on which to test the tips
- Variability **between** blocks can be large, variability **within** a block should be relatively small
- In general, a **block** is a specific level of the nuisance factor
- A complete replicate of the basic experiment is conducted in each block
- A block represents a **restriction on randomization**
- All runs **within** a block are **randomized**

The Hardness Testing Example

- Suppose that we use $b = 4$ blocks:

Table 4-1 Randomized Complete Block Design
for the Hardness Testing Experiment

Test Coupon (Block)			
1	2	3	4
Tip 3	Tip 3	Tip 2	Tip 1
Tip 1	Tip 4	Tip 1	Tip 4
Tip 4	Tip 2	Tip 3	Tip 2
Tip 2	Tip 1	Tip 4	Tip 3

- Notice the **two-way structure** of the experiment
- Once again, we are interested in testing the equality of treatment means, but now we have to remove the variability associated with the nuisance factor (the blocks)

Extension of the ANOVA to the RCBD

- Suppose that there are a treatments (factor levels) and b blocks
- A **statistical model** (effects model) for the RCBD is

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, b \end{cases}$$

- The relevant (fixed effects) hypotheses are

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_a \text{ where } \mu_i = (1/b) \sum_{j=1}^b (\mu + \tau_i + \beta_j) = \mu + \tau_i$$

Extension of the ANOVA to the RCBD

ANOVA partitioning of total variability:

$$\begin{aligned}\sum_{i=1}^a \sum_{j=1}^b (y_{ij} - \bar{y}_{..})^2 &= \sum_{i=1}^a \sum_{j=1}^b [(\bar{y}_{i.} - \bar{y}_{..}) + (\bar{y}_{.j} - \bar{y}_{..}) \\ &\quad + (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})]^2 \\ &= b \sum_{i=1}^a (\bar{y}_{i.} - \bar{y}_{..})^2 + a \sum_{j=1}^b (\bar{y}_{.j} - \bar{y}_{..})^2 \\ &\quad + \sum_{i=1}^a \sum_{j=1}^b (y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..})^2 \\ SS_T &= SS_{Treatments} + SS_{Blocks} + SS_E\end{aligned}$$

Extension of the ANOVA to the RCBD

The degrees of freedom for the sums of squares in

$$SS_T = SS_{Treatments} + SS_{Blocks} + SS_E$$

are as follows:

$$ab - 1 = a - 1 + b - 1 + (a - 1)(b - 1)$$

Therefore, ratios of sums of squares to their degrees of freedom result in mean squares and the ratio of the mean square for treatments to the error mean square is an F statistic that can be used to test the hypothesis of equal treatment means

ANOVA Display for the RCBD

Table 4-2 Analysis of Variance for a Randomized Complete Block Design

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	$SS_{\text{Treatments}}$	$a - 1$	$\frac{SS_{\text{Treatments}}}{a - 1}$	$\frac{MS_{\text{Treatments}}}{MS_E}$
Blocks	SS_{Blocks}	$b - 1$	$\frac{SS_{\text{Blocks}}}{b - 1}$	
Error	SS_E	$(a - 1)(b - 1)$	$\frac{SS_E}{(a - 1)(b - 1)}$	
Total	SS_T	$N - 1$		

Manual computing (ugh!)...see Equations (4-9) – (4-12), page 124

Design-Expert analyzes the RCBD

Vascular Graft Example (pg. 124)

- To conduct this experiment as a RCBD, assign all 4 pressures to each of the 6 batches of resin
- Each batch of resin is called a “**block**”; that is, it’s a more homogenous experimental unit on which to test the extrusion pressures

Table 4-3 Randomized Complete Block Design for the Vascular Graft Experiment

Extrusion Pressure (PSI)	Batch of Resin (Block)						Treatment Total
	1	2	3	4	5	6	
8500	90.3	89.2	98.2	93.9	87.4	97.9	556.9
8700	92.5	89.5	90.6	94.7	87.0	95.8	550.1
8900	85.5	90.8	89.6	86.2	88.0	93.4	533.5
9100	82.5	89.5	85.6	87.4	78.9	90.7	514.6
Block Totals	350.8	359.0	364.0	362.2	341.3	377.8	$y_{..} = 2155.1$

Vascular Graft Example

Design-Expert Output

Response: Yield

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Block	192.25	5	38.45		
Model	178.17	3	59.39	8.11	0.0019
A	<i>178.17</i>	<i>3</i>	<i>59.39</i>	<i>8.11</i>	<i>0.0019</i>
Residual	109.89	15	7.33		
Cor Total	480.31	23			
Std. Dev.	2.71			R-Squared	0.6185
Mean	89.80			Adj R-Squared	0.5422
C.V.	3.01			Pred R-Squared	0.0234
PRESS	281.31			Adeq Precision	9.759

Residual Analysis for the Vascular Graft Example

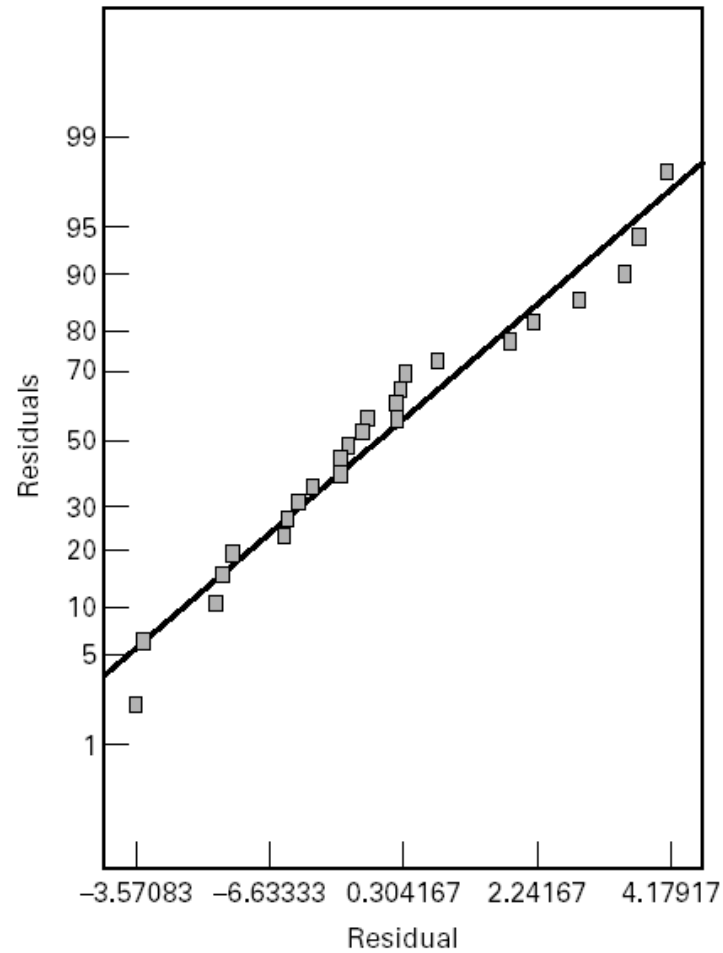


Figure 4-4 Normal probability plot of residuals for Example 4-1.

Residual Analysis for the Vascular Graft Example

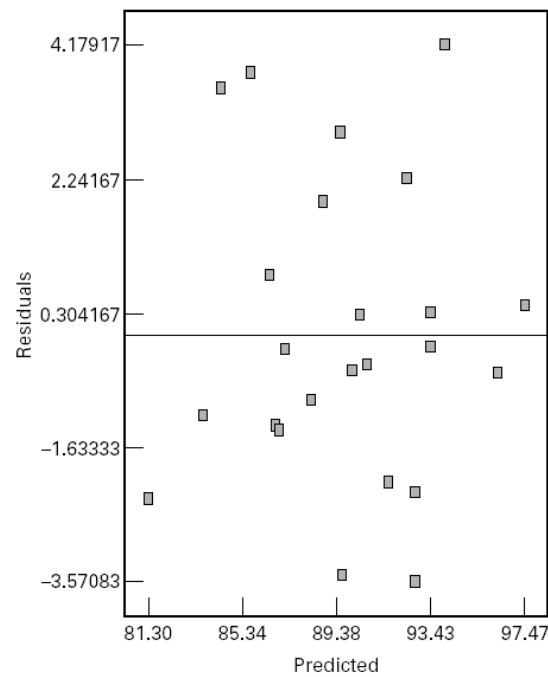


Figure 4-5 Plot of residuals versus \hat{y}_i for Example 4-1.

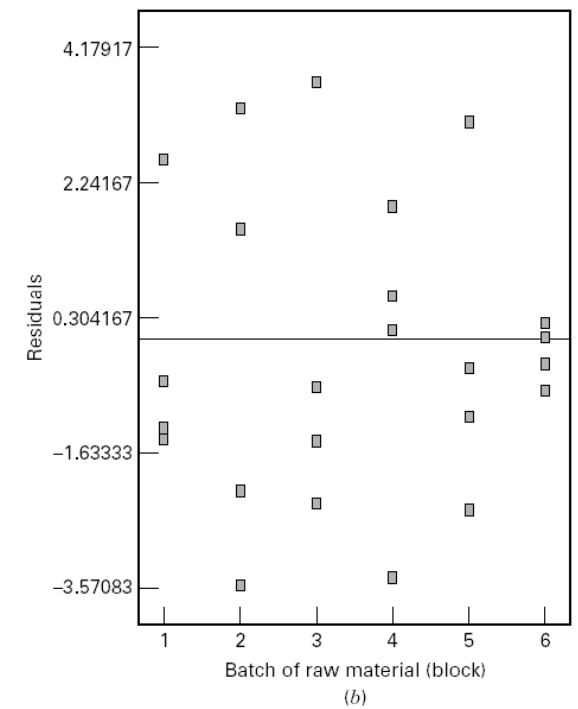
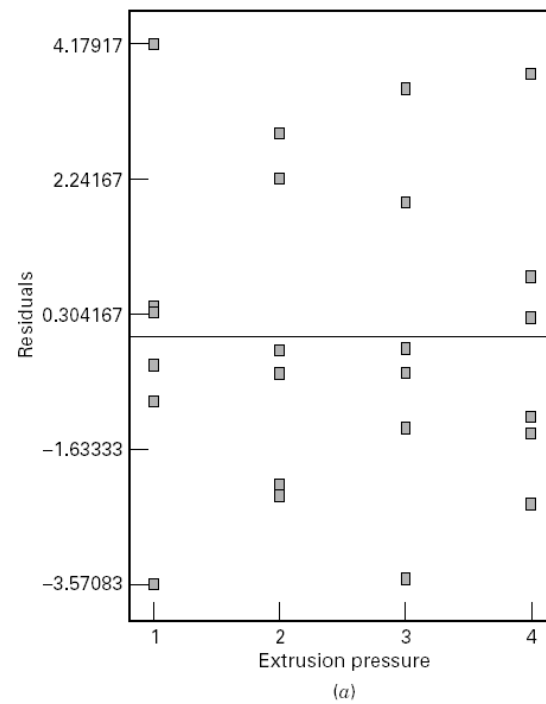


Figure 4-6 Plot of residuals by extrusion pressure (treatment) and by batches of resin (block) for Example 4-1.

Residual Analysis for the Vascular Graft Example

- Basic residual plots indicate that **normality**, **constant variance** assumptions are satisfied
- No obvious problems with **randomization**
- **No patterns in the residuals vs. block**
- Can also plot residuals versus the pressure (residuals by factor)
- These plots provide more information about the constant variance assumption, possible outliers

Multiple Comparisons for the Vascular Graft Example – Which Pressure is Different?

Treatment Means (Adjusted, If Necessary)					
	Estimated Mean		Standard Error		
1-8500	92.82		1.10		
2-8700	91.68		1.10		
3-8900	88.92		1.10		
4-9100	85.77		1.10		
Treatment	Mean Difference	DF	Standard Error	t for H₀ Coeff=0	Prob > t
1 vs 2	1.13	1	1.56	0.73	0.4795
1 vs 3	3.90	1	1.56	2.50	0.0247
1 vs 4	7.05	1	1.56	4.51	0.0004
2 vs 3	2.77	1	1.56	1.77	0.0970
2 vs 4	5.92	1	1.56	3.79	0.0018
3 vs 4	3.15	1	1.56	2.02	0.0621

Also see Figure 4-3, Pg. 128

Other Aspects of the RCBD

See Text, Section 4-1.3, pg. 130

- The RCBD utilizes an **additive model** – no interaction between treatments and blocks
- Treatments and/or blocks as random effects
- Missing values
- What are the **consequences** of **not blocking** if we should have?
- **Sample sizing** in the RCBD? The **OC curve** approach can be used to determine the number of blocks to run..see page 131

The Latin Square Design

- Text reference, Section 4-2, pg. 136
- These designs are used to simultaneously control (or eliminate) **two sources of nuisance variability**
- A significant assumption is that the three factors (treatments, nuisance factors) **do not interact**
- If this assumption is violated, the Latin square design will not produce valid results
- Latin squares are not used as much as the RCBD in industrial experimentation

The Rocket Propellant Problem – A Latin Square Design

Table 4-8 Latin Square Design for the Rocket Propellant Problem

Batches of Raw Material	Operators				
	1	2	3	4	5
1	$A = 24$	$B = 20$	$C = 19$	$D = 24$	$E = 24$
2	$B = 17$	$C = 24$	$D = 30$	$E = 27$	$A = 36$
3	$C = 18$	$D = 38$	$E = 26$	$A = 27$	$B = 21$
4	$D = 26$	$E = 31$	$A = 26$	$B = 23$	$C = 22$
5	$E = 22$	$A = 30$	$B = 20$	$C = 29$	$D = 31$

- This is a 5×5 Latin square design
- Page 140 shows some other Latin squares
- Table 4-13 (page 140) contains properties of Latin squares
- Statistical analysis?

Statistical Analysis of the Latin Square Design

- The statistical (effects) model is

$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \begin{cases} i = 1, 2, \dots, p \\ j = 1, 2, \dots, p \\ k = 1, 2, \dots, p \end{cases}$$

- The statistical analysis (ANOVA) is much like the analysis for the RCBD.
- See the ANOVA table, page 137 (Table 4-9)
- The analysis for the rocket propellant example is presented on text pages 138 & 139